18.727, Topics in Algebraic Geometry (rigid analytic geometry)
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More corrections on Berkovich spaces

I'll get this right eventually, I promise. (Again, many thanks to Brian Conrad for clarifications; remaining errors are of course not his fault!)

The definition of a quasi-separated rigid space is not what I said in class. It is (just as for schemes): X is quasi-separated if $X \to \operatorname{Max} K$ is quasi-separated, and a morphism $X \to Y$ is quasi-separated if the diagonal $\Delta_{X/Y} : X \to X \times_Y X$ is a quasi-compact morphism (not a locally closed immersion). Oh, and a morphism is quasi-compact if the inverse image of every quasi-compact open is quasi-compact.

Re the notion of a net: the restriction of a quasi-net to an open subset means take elements of the quasi-net contained in the open, rather than intersecting all of them with the open. So my example with the unit squares is not a net; you can fix it by adding in the closed edges and the corners.

More serious functor: the rigid-to-Berkovich functor is backwards from what I've been saying! Mea culpa. It turns Hausdorff strictly analytic spaces into quasiseparated rigid spaces, and in *that* direction is fully faithful. It is true, though, that it induces an equivalence between paracompact strictly analyic spaces and quasi-separated rigid spaces with an admissible affinoid covering of finite type. ("Finite type" means that any element of the covering meets only finitely many others. E.g., the covering of the open unit disc by closed discs around the origin is not of finite type, but there is an admissible covering of finite type by closed annuli.) The point here is that quasi-separatedness is needed in order to glue meaningfully (otherwise the attaching maps on the Berkovich affinoids are not uniquely determined by their rigid counterparts, so the cocycle condition falls apart) and the finite type condition is needed in order to build a quasi-net. (Maybe with a more sensible definition of Berkovich's spaces you could get past the finite type issue?) Anyway, see Theorem 1.6.1 of Berkovich's IHES paper.

Upshot: the Berkovich category neither contains nor is contained in the category of rigid spaces, but they share the rigid spaces which are "not too pathological", which the spaces you typically encounter in practice will be.

And one fun thing you might want to try (suggested by Berkovich in his ICM talk): compute the Gelfand spectrum of $\mathbb{Z}$. (Remember, this means you allow the normal triangle inequality, and you don't impose any upper bound on the seminorms.)