Math 204B: Number Theory
UCSD, winter 2017
Problem Set 5 (due *Friday*, March 17)

1. Give an example of a Galois extension $L/\mathbb{Q}$ of number fields such that for some prime p, the decomposition groups of the extensions of the p-adic valuation to L are not normal subgroups of $\mathrm{Gal}(L/\mathbb{Q})$.

2. Suppose that K is a finite extension of $\mathbb{Q}_2$ and that L/K is a finite Galois extension of degree 8 whose Galois group is the quaternionic group

$$G = \langle i, j, k | i^2 = j^2 = k^2 = ijk = 1 \rangle.$$

Suppose further that L/K is totally ramified and that $G_4 = 1$.

   (a) Show that $G_2 = G_3 = \{\pm 1\}$, the center of G.
   (b) Prove that the residue field of K must be strictly larger than $\mathbb{F}_2$.
   (c) Prove that $\frac{3}{2}$ is a ramification break for G in the upper numbering.

3. Find an explicit example which satisfies the hypotheses of the previous problem with K equal to the unramified quadratic extension of $\mathbb{Q}_2$. Resources you might (or might not) find helpful include computer algebra systems that support algebraic number theory (e.g., *Pari*, *Sage*, or *Magma*), and the L-Functions and Modular Forms Database (`http://lmfdb.org`).

4. CFT notes, exercise 1.2.

5. CFT notes, exercise 3.5.

6. CFT notes, exercise 3.6.

7. CFT notes, exercise 4.4.