A–1 A right circular cone has base of radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube?

A–2 Let be any arc of the unit circle lying entirely in the first quadrant. Let be the area of the region lying to the right of the -axis and to the left of . Prove that depends only on the arc length, and not on the position, of .

A–3 Let be a real function on the real line with continuous third derivative. Prove that there exists a point such that
\[ f(a) \cdot f'(a) \cdot f''(a) \cdot f'''(a) \geq 0. \]

A–4 Let and . For , the number is defined by concatenating the decimal expansions of , and from left to right. For example , , , , and so forth. Determine all such that divides .

A–5 Let be a finite collection of open discs in whose union contains a set . Show that there is a pairwise disjoint subcollection such that
\[ E \subseteq \bigcup_{j=1}^{n} 3D_j. \]
Here, if is the disc of radius and center , then is the disc of radius and center .

A–6 Let denote distinct points with integer coordinates in . Prove that if
\[ (|AB| + |BC|)^2 < 8 \cdot |ABC| + 1 \]
then , , are three vertices of a square. Here is the length of segment and is the area of triangle .

B–1 Find the minimum value of
\[ \frac{(x + 1/x)^6 - (x^6 + 1/x^6) - 2}{(x + 1/x)^3 + (x^3 + 1/x^3)} \]
for .

B–2 Given a point with , determine the minimum perimeter of a triangle with one vertex at , one on the -axis, and one on the line . You may assume that a triangle of minimum perimeter exists.

B–3 Let be the unit hemisphere and the unit circle . The regular pentagon inscribed in lies over the planar region inside . Write your answer in the form , where , , are real numbers.

B–4 Find necessary and sufficient conditions on positive integers and so that
\[ \sum_{n=0}^{m-1} (-1)^{|i/m|+|i/n|} = 0. \]

B–5 Let be the positive integer with decimal digits, all of them . That is,
\[ N = 1111 \cdots 11. \]
Find the thousandth digit after the decimal point of .

B–6 Prove that, for any integers , , there exists a positive integer such that is not an integer.