

18.726 Problem Set 3, due Thursday, February 24

Please submit *eight* of the following problems, including all problems marked “Required”. This would be a good time to point out that I do consider it acceptable to invoke the statement of one problem from the book in the solution of another problem, even if you never submitted the first one, or even if I never assigned it. But you should feel at least a little twinge of guilt about doing so! (Only a little one, though.)

1. (Required)
 - (a) Prove that the property of a morphism being locally of finite type/presentation is local on the base (cf. Hartshorne II.3.1).
 - (b) Prove that a morphism is of finite type/presentation if and only if it is locally of finite type/presentation and quasi-compact (cf. Hartshorne II.3.3(a)).
 - (c) Deduce (using II.3.2) that the property of morphism being of finite type/presentation is local on the base.
2. Hartshorne II.3.4.
3. Hartshorne II.3.5.
4. Hartshorne II.3.7.
5. Hartshorne II.3.8. Not required, but maybe it should be.
6. Hartshorne II.3.14, first part; the second part is optional but try it anyway.
7. Hartshorne II.3.15. Hint: the simplest example of something which is reduced but not geometrically reduced is $\text{Spec } k[x]/(x^p - c)$, where k is a field of characteristic $p > 0$ which is not perfect, and $c \in k$ is not a p -th power.
8. Hartshorne II.3.19 (not for the faint of heart).
9. Read on your own what dimension means (page 86), then do either of Hartshorne II.3.20 or II.3.21. Actually, please do the reading even if you don't submit this.
10. (Required) Hartshorne II.4.1. (Hint: use exercise II.3.4.)
11. Hartshorne II.4.4.
12. Hartshorne II.4.6.
13. (Required) Hartshorne II.4.8.