18.726 Problem Set 4, due Thursday, March 3

Please submit *six* of the following problems. (The set is short this week because we only had one lecture since PS3 came due.) For the ones marked "Required if you didn't do it last time", if you did do them on PS3, don't do them again!

1. (Required if you didn't do it last time) Hartshorne II.4.1. (Hint: use exercise II.3.4.)
2. Hartshorne II.4.5.
3. Hartshorne II.4.7.
4. (Required if you didn't do it last time) Hartshorne II.4.8.
5. Hartshorne II.4.11.
6. Hartshorne II.5.2.
7. Hartshorne II.5.5.
8. Hartshorne II.6.10.
9. Hartshorne II.7.1.
10. (Required) Let k be an algebraically closed field. Classify up to isomorphism the closed subschemes Z of $\operatorname{Spec} k[x, y]$ of dimension 0 and degree 3 over k. Here "dimension 0 and degree 3" means that $\operatorname{Spec} k[x, y]/I$ is a three-dimensional k-vector space.
11. (Even if you don't submit this one, make sure you understand what it says!) Let k be a field, and let $\mathcal{F}$ be the contravariant functor that associates to a k-scheme X the set of closed subschemes Y of $\mathbb{P}^2_k \times_k X$ such that $\mathrm{pr}_2 : Y \to X$ is finite and $(\mathrm{pr}_2)_* \mathcal{O}_Y$ is a locally free sheaf of rank 3 on X. (The functoriality here converts $X' \to X$ to the pullback from $\mathbb{P}^2_K \times_k X$ to $\mathbb{P}^2_k \times_k X'$. Also note that I want the actual set, *not* the set of isomorphism classes as in the previous exercise!) Prove that $\mathcal{F}$ is representable; in other words, there is a k-scheme U and an element Y_U of $\mathcal{F}(U)$ such that for each k-scheme X, we can describe $\mathcal{F}(X)$ as $\operatorname{Hom}(X, U)$ by identifying $f : X \to U$ with $f^* Y_U$. (In other words, Y_U is the "Hilbert space of subschemes of $\mathbb{P}^2_k$ of dimension 0 and degree 3." This construction is not special to the parameters 0 and 3, but the proof is more complicated in general.)