18.787: Topics in Number Theory (p-adic differential equations)
Kiran Kedlaya, fall 2007

The p-adic numbers, which are produced by completing the field of rational numbers using the p-adic absolute value instead of the ordinary (archimedean) absolute value, play a fundamental role in modern number theory. In particular, one often wishes to carry analytic techniques from the real/complex field to the p-adics; this course will discuss a particular situation where this becomes quite complicated/interesting, namely the theory of ordinary differential equations.

The fact that ODEs over the p-adics have any meaning for number theory was originally discovered by Dwork, who discovered that in many cases, one could use solutions of certain p-adic differential equations to compute numbers of solutions of certain polynomial equations mod p (or more generally, zeta functions of algebraic varieties over finite fields). Moreover, the equations that arose tended to be closely related to classical Fuchsian equations (hypergeometric, Bessel, etc.).

This course will be a survey of the resulting theory of p-adic differential equations, including some applications to variation of zeta functions. I'll try to trace a path between Dwork's original point of view, which was very much oriented towards specific examples and explicit computations, and the more modern viewpoint that has evolved subsequently, in an attempt to make the subject more systematic, with more widely applicable theorems.

The primary prerequisite is being comfortable with the field of p-adic numbers; I'll review such concepts as Newton polygons, ramification theory, and such at the beginning, but I'm presuming you saw them somewhere, like in 18.786. You should also be happy with one-variable complex analysis (18.112). Some familiarity with algebraic geometry (18.725 should suffice) will be helpful in a few places.

Recommended text: Dwork, Gerotto, and Sullivan, *An Introduction to G-Functions* (Princeton University Press).

For more information, see the course web site:

`http://math.mit.edu/~kedlaya/18.787/`

(Note on the subject number: 18.787 is a newly created number for advanced topics in number theory. The instructor, topic, prerequisites, and even whether this subject is offered at all, will vary from year to year.)