1. (a) Yes, the map is injective: every holomorphic function on an open disc is represented by its Taylor series.
   
   (b) No, the map is not injective: the function $e^{-1/x^2}$ is $C^\infty$ but its Taylor series is identically zero.

2. Suppose that $s_1, s_2 \in F(U)$ maps to the same element $\prod_{x \in U} F_x$. For each $x \in X$, we can find an open neighborhood $U_x$ of $x$ in $U$ such that the restrictions of $s_1$ and $s_2$ to $U_x$ coincide. Since the sets $U_x$ form a covering of $U$, we deduce from the sheaf axiom that $s_1 = s_2$.

   For an example of a presheaf where this fails, take for instance $F$ to be the sheaf on $\mathbb{R}$ such that
   
   $$
   F(U) = \begin{cases}
   \mathbb{R} & U = \mathbb{R} \\
   \{0\} U \neq \mathbb{R}.
   \end{cases}
   $$

   Then $F_x = 0$ for all $x \in \mathbb{R}$, so the map $F(\mathbb{R}) \to \prod_{x \in U} F_x$ cannot be injective.

3. A left adjoint to the forgetful functor $\textbf{Ab} \to \textbf{Set}$ is given by the functor taking a set $S$ to be the free abelian group on generators indexed by the elements of $S$. Similarly, a left adjoint to the forgetful functor $\textbf{Ring} \to \textbf{Set}$ is the functor taking the set $S$ to the polynomial ring over $\mathbb{Z}$ with variables indexed by the elements of $S$.

4. The generic point $(0)$ has a unique preimage $(0)$. The point $(p)$ has one preimage if $p = 2$ or $p \equiv 3 \pmod{4}$, and two preimages if $p \equiv 1 \pmod{4}$.

5. The closed points correspond to individual real numbers and to conjugate pairs of nonreal complex numbers.

6. We may write $U$ as the union of the distinguished open subsets $D(2)$, $D(x)$, which intersect in $D(2x)$; we have
   
   $$
   D(2) = \text{Spec } \mathbb{Z}[x][1/2], \quad D(x) = \text{Spec } \mathbb{Z}[x][1/x], \quad D(2x) = \text{Spec } \mathbb{Z}[x][1/2, 1/x].
   $$

   The sections $\mathcal{O}_X(U)$ comprise the intersection of the first two rings inside the third one, which is exactly $\mathbb{Z}[x]$.

   If $U$ were affine, it would then equal $\text{Spec } \mathbb{Z}[x]$. By a theorem from class, the inclusion $U \to X$ would then arise from a map $\mathcal{O}_X(X) \to \mathcal{O}_U(U)$. But the source and the target of this map are $\mathbb{Z}[x]$; by restricting to $D(2)$, we see that the map $\mathbb{Z}[x] \to \mathbb{Z}[x]$ we get is the identity map. But $U \to X$ is not an isomorphism, contradiction.

7. We claim that the quotient bijects to $\text{Spec}(R)$. Note that if $R \to F_1, R \to F_2$ are equivalent, then $\ker(R \to F_1) = \ker(R \to F_2)$ are the same prime ideal of $R$; this gives the map to $\text{Spec}(R)$.
To check that the map is surjective, note that every prime ideal $p$ arises as the kernel of $R \to \text{Frac}(R/p)$.

To check that the map is injective, suppose that $\ker(R \to F_1) = \ker(R \to F_2) = p$. Put $F_0 = \text{Frac}(R/p)$; then $F_0$ maps to $F_1$ and $F_2$, necessarily injectively (field homomorphisms are always injective). Take the ring $F_1 \otimes_{F_0} F_2$ and quotient by any maximal ideal; we then get a field $F_3$ and homomorphisms $F_1 \to F_3, F_2 \to F_3$ such that the compositions $R \to F_1 \to F_3, R \to F_2 \to F_3$ coincide.