Math 203B (Algebraic Geometry), UCSD, winter 2020
Problem Set 8 (due *Friday*, March 13)

Solve the following problems, and turn in the solutions to at least *four* of them.

1. Hartshorne, exercise II.6.6.

2. Let X be the scheme $\operatorname{Spec} \mathbb{C}[x, y]$. Among the blowups of X along the ideals

$$(x, y), (x, y^2), (x^2, y^2), (x^2, xy, y^2), (x^2, xy),$$

   determine which ones are isomorphic. (Hint: see Hartshorne, exercise II.7.11.)

3. Hartshorne, exercise II.7.12.

4. Use Hartshorne, example II.8.20.1 (which works over any base ring, not necessarily a field) and a previous assignment to compute the cohomology of the sheaf $\omega_X(d)$ where $X = \mathbb{P}^n_A$ for some ring A.

5. Hartshorne III.4.7.

6. Put $X = \operatorname{Spec} \mathbb{C}[x_1, x_2, x_3, x_4]$, let Y_1 be the plane $x_1 = x_2 = 0$ in X, and let Y_2 be the plane $x_3 = x_4 = 0$ in X. Compute Čech cohomology of $U = X \setminus (Y_1 \cup Y_2)$ with respect to the open covering

$$D(x_1x_3), D(x_1x_4), D(x_2x_3), D(x_2x_4)$$

   to see that $H^2(U, \mathcal{O}_U) \neq 0$. (This is one step of Hartshorne, exercise III.4.9.)