

Plan for today:

Course website

- Welcome all!

Canvas

- Tech tools (Zoom, Zulip, Miro, CoCalc)

- Syllabus

~ Facilitators

- Get started on the material

(as time permits)

Homework due Thursdays

(weeks 2-7, 9-10)

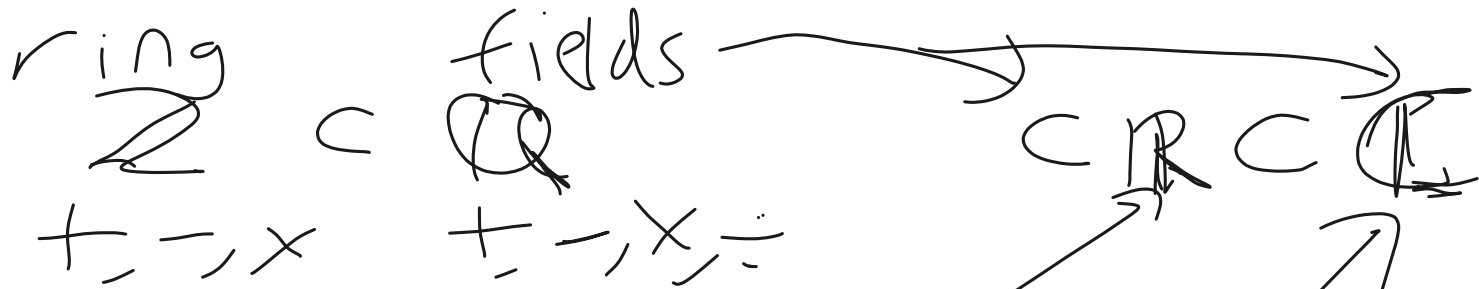
Submitted via CoCalc
drop 1 homework

Collaboration/research encouraged

but submit solutions in your own
words
and cite

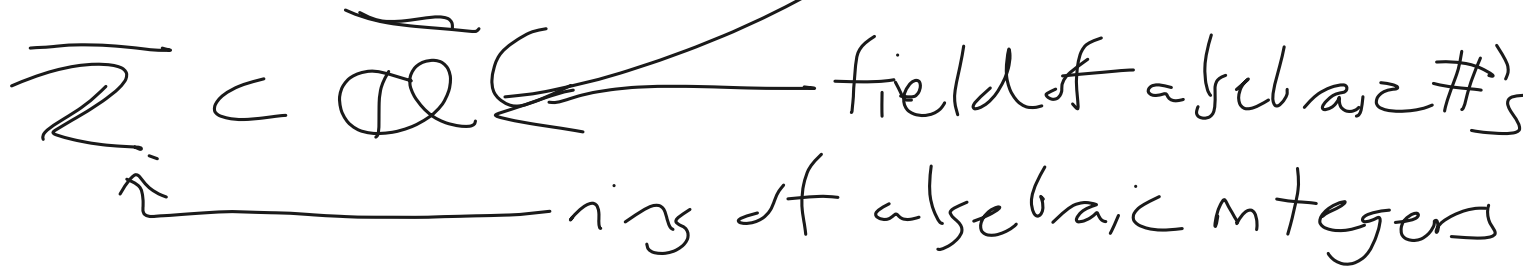
Facilitators:

- message me on Zulip
- separate stream on Zulip
- separate office hours



$$a^2 + b^2 = c^2 \quad \sqrt{2}$$

(3, 4, 5)



Fundamental Theorem of Algebra

Every ^{non zero} polynomial $P(x) \in \mathbb{Q}[x]$
factors as a product of --
a constant times some ^{monic} linear
polynomials, uniquely up to order.

$$P(x) = c(x - \underline{r_1}) \cdots (x - \underline{r_n})$$

An element α of \mathbb{C} is an algebraic number if there exists a ^{non-zero} polynomial

$P(x) \in \mathbb{Q}[x]$ such that $P(\alpha) = 0$.

e.g. any rational, $\sqrt{2}$, i , ...

but not π , e (hard theorems)

Moreover, most numbers are not algebraic

$$a^2 + b^2 = c^2$$

$$(a + bi)(a - bi) = c^2$$

leading
coeff = 1

A number $\alpha \in \mathbb{C}$ is an algebraic integer if there is a monic poly

$P(x) \in \mathbb{Z}[x]$ such that $P(\alpha) = 0$

e.g. \mathbb{Z} , i , $\sqrt{2}$, $a + bi$ ($a, b \in \mathbb{Z}$)

not examples: π, e

$$x^2 + 1/4$$

$$i/2, \mu$$

~~$$4/x^2 + 1$$~~

equivalent condition:

if $\alpha \in \mathbb{C}$ is an algebraic number

$\exists!$ monic poly $P(x) \in \mathbb{Q}[x]$ of minimum degree

for which $P(\alpha) = 0$. Then α is an algebraic

integer iff this $P(x)$ is in $\mathbb{Z}[x]$.