Plan for today:

- Welcome all!
- Tech tools (Zoom, Zulip, Miro, CoCalc)
- Syllabus
- Facilitators
- Get started on the material (as time permits)

Course website
Canvas

Homework due Thursdays
(weeks 2–7, 9–10)
submitted via Cocalc
drop 1 homework
collaboration/research encouraged
but submit solutions in your own words
and cite

Facilitators:

- message me on Zulip
- separate stream on Zulip
- separate office hours

ring fields

$\mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$

$+, -, \times \qquad +, -, \times, \div$

$a^2+b^2=c^2 \qquad \sqrt{2}$

$(3, 4, 5)$

$\overline{\mathbb{Z}} \subset \overline{\mathbb{Q}} \subset$

field of algebraic #'s

rings of algebraic integers

Fundamental Theorem of Algebra

Every nonzero polynomial $P(x) \in \mathbb{C}[x]$ factors as a product of a constant times some monic linear polynomials, uniquely up to order.

$$P(x) = c(x - r_1) \cdots (x - r_n)$$

An element α of $\mathbb{C}$ is an <u>algebraic number</u> if there exists a nonzero polynomial $P(x) \in \mathbb{Q}[x]$ such that $P(\alpha) = 0$.

e.g. any rational, $\sqrt{2}$, i, ...

but not π, e (<u>hard</u> theorems)

Moreover, <u>most</u> numbers are not algebraic

$a^2+b^2=c^2$

$(a+bi)(a-bi)=c^2$

A number $\alpha \in \mathbb{C}$ is an <u>algebraic integer</u> if there is a <u>monic</u> poly $P(x) \in \mathbb{Z}[x]$ such that $P(\alpha)=0$

leading coeff $= 1$

e.g. $\mathbb{Z}$, i, $\sqrt{2}$ $a+bi$ $(a,b \in \mathbb{Z})$

nonexamples: π, e $\quad x^2 + 1/4$

$i/2$ $\quad \cancel{4/x^2+1}$

equivalent condition:

if $\alpha \in \mathbb{C}$ is an algebraic number, $\exists!$ monic poly $P(x) \in \mathbb{Q}[x]$ of <u>minimum</u> degree for which $P(\alpha)=0$. Then α is an algebraic integer <u>iff</u> this $P(x)$ is in $\mathbb{Z}[x]$.