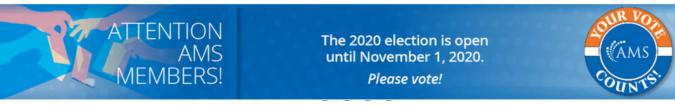
## **Ideals and factorization in number fields**

Happy Indigenous Peoples' Day!

**Note:** the video from October 9 got corrupted while I was trying to post it, so there is a segment missing in the middle. I recorded a supplemental video to cover the gap.



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## The trace of an algebraic integer

Reninver for 4/K Finite Fieldextersion Trace L->K Trace (X)=Irace (Tx. L->L) What IF K=Q, XEQ\_? The Trace (X) EQ DE= 2.

The trace pairing L/K frik tieldesterion For X. Y EL, defne (X, y) = trace (Xy) 4/K K-bilmer map LXKL-)K Prop if YK is separable (and d,... h 13 a basis of L/K) then (X, 7) is (perfection fire L ~ L ~ L ~ Man (5K) PE whe L=K(0), se 6-1, 1,0... 017 (any te det it pairis matrix (Vendemode)

Integral bases Carollary if Lisa number field then of and = ELiAD OL is finite free Enother (it. a Infitice in Las a Revector space). PE Pick a basis of La consisting of elements of OL This gives a ype band on OL 6/2 painty 15 perfect.

Ideals in rings of integers L = # Held2×3=(1+1-5)(1-1-5) in 245) Kimmer: Fix this by adding "ideal numbers" reged(2,1+r-5) Dedekad instead, consider set at milityee R=my ICRISE del if = XYEI = XYEI Prennslik ) VER ) xEI YER ) xJEZ VER ) XEI YER ) XJEZ

**Dedekind domains** An ntegal doman Risa Dedekind domain it: ~ K is northerian · Ris stephly clused in FracER) · Every nonzero price ilen is maximal Recall: ICR is prime if VxyER, if XYET ideal then XILI on YET.  $(e_{1}, \mathbb{Z})$ 

<u>**Rings of integers are Dedekind domains**  $\mathcal{L} = \mathcal{T} + field$ </u> Thearen OL, is a Dedekind dancin. 400 - noetherin (e, Norminal) megally closed (e, Norminal) let I kanonzer me ikal. The InZ £ 63 15 a nonzer proceideal say (p). The ring Oken is finite are Rep= Fp & is integal doman, so a field  $X' + a_{\mu} X^{1} + - + q_{\mu} = 0 - q_{\mu} = X(X' + q_{\mu} X' + q_{\mu})$ So I maximal

**Statement of unique factorization in a Dedekind domain** The oren For R a Pedekind domain, every nonzer ickal I adnits a fu christianas from fra ubere porcers Pi fraepsme ideal, mathfrak which is might up to arte (Note: no vits!)

A lemma on products of prime ideals Ribedeund Lemma: Farley more ideal I dema 'MR' ∃ f, frpriveileals sit I = f, -fr Pf otherise find a maximal conterraple which cannot be prove 10 7 XIXER sit XYET, WHXTTYTT FI YINK- 7, 2 +, Prg. 25.