## The Kronecker-Weber theorem: a preview of Math 204B

Last lecture this Friday (December 11). Regular office hours also end this week, but I plan to hold some sporadic office hours after that (see Zulip).

Course evaluations due Monday, December 14.

I can accept late homeworks through Friday, December 18. (Grades are due December 22.)

Math 204B begins Monday, January 4 (new home page, same Zoom, same Zulip).

This lecture and the next are based on my notes on class field theory: https://kskedlaya.org/cft.

The Kronecker-Weber theorem: statement The let K le a runte Held which is Galasone Q who bel(K/Q) abelia ( i.e. K is a <u>alelian exterior</u> of (e) The K C R ( gn) for some n. C.S. Every gudate extension has this papety (Gavis)

## Artin maps for abelian number fields

let K/ & be mabel mexterion. By K-W. KEGl(gn) for some n. ale 1.2m (Z/R) = Gal (Q(gn)/Q) ) Gal(K/Q) der nor a 1 ~ (G H) ya) let p beja ratural prove not dividing n. The privator mility on Q(gn) or K, Spick a prove of other clone p; her Gp is give guested by Fill f. Arter reciprocity lan for the.

<u>A further preview: the Artin reciprocity law</u> Fighis (livening, let Lik ag abelin extension of Nomber fields Altin recipility law: there is a similar relationship uchen Gal (LIK) and sure group utiles classes which predicts (i.e. some grother to F JK Fodenius elements for defined by a congrence unsched ports condition on prisped ident) Folphics derents for unranified norths of K

A further preview: the existence theorem The existence there all assort that the conjustitum of all abelia extensions of K 15'il big as possible sine AAn receptiony! huve instruct it will at be men enjoint (K=includy Judzhe - set explicit alelia extensions for (Melliphi K=fuctor fields - set adelin extensions for Druted moder) K = local field - Libm - Take the any

## **The local Kronecker-Weber theorem: statement**

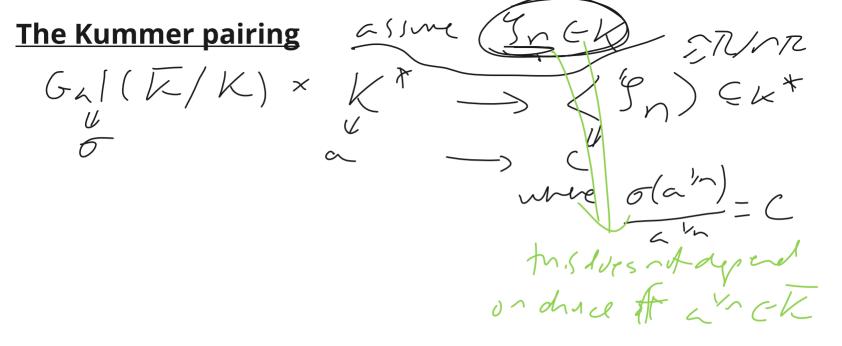
The let k/ Aly lex toute abelin extension. The KGlep(5) for some n.

(There is a analyze of AAn regrand, by it's not get den han to furnulate t.) better furnulation will be via a deles

Reduction to the local case The Local Kw (brall) =)K-W. It let k be nabelin esterson it De The let k be have more that milder in K, for each prime post & that milder in K, apply local K w to inter ke E (Ep(Ging)) (At E sure Ame above p) (et n = Tp proting) we claim K E (R/Sm) it. K(Sm) = (R/Sm)) L =

Reduction to the local case L=K(Yn) neth support Jp= meta support p, n L a protot-L abure p) let U= morral unraited exterior.t F. vp(n)  $= I_{\mathcal{Z}} one (lp. The U(S_{p*})=L_{\mathcal{Z}})$   $= J_{\mathcal{Z}} = L_{\mathcal{U}}(L_{\mathcal{Z}}/\mathcal{U}) \in [\mathcal{D}/\mathcal{V}^{(n)}]^{*} (U_{\mathcal{U}}) \times kral - \mathcal{U}$   $= M_{\mathcal{Z}}(\mathcal{U}) = (\mathcal{U}) = (\mathcal{U}) \times ($ 4+ I be the supperented in all of the Ips the  $|I| \leq \prod |I| \leq \prod |I| \leq \prod p(p^{m(n)}) = p(n) = [(p(M))]$ mennhile, fræd teld of I is ensubere manked ig  $(\mathcal{M}_{nh}, \mathcal{M}_{n}) = (\mathcal{L} \cdot \mathcal{U}) \leq (\mathcal{P}(\mathcal{M}_{n}) \cdot \mathcal{U}) = \mathcal{L} = \mathcal{P}(\mathcal{S}_{n}),$ 

Kummer extensions let ~ be apositive interest let Kbe a feld of chracterite not diding n. For eny ack, K(~") is sure held estern I latk when a Norlet's assume gnEK provider of sut of it. Then K(a'n) is a balais extension of K ut Gal jour some stymp of ZINZ (ELNR)\* - (G' -) a' yn) (+ Gal(K(a'n)/K)=Ring)() a((K\*)) to ever proceeding for a factor pot n.



<u>Kummer's theorem (statement) and "Kummer theory"</u> There me Kinne guring indrees in Tsimeform K\* (K\*) ~ = Mon (Gal(K/K), (4)) (K\*) ~ m uhich Kicher Khurgin Gal(L/K) (if. (entrops for produite Nydry of Gl(K/K) (y, n=2)