

**Math 204B (Algebraic Number Theory), UCSD, winter 2015**  
**Problem Set 6 (due Wednesday, February 25)**

Solve the following problems, and turn in the solutions to *four* of them. Note: in this and subsequent problem sets, CFT refers to my lecture notes on class field theory: <http://math.ucsd.edu/~kedlaya/math204b/cft-overall.pdf>.

1. Neukirch, Exercise III.2.1 (including the counterexample, for which you may use Sage if you wish).
2. Neukirch, Exercise III.2.3.
3. Neukirch, Exercise III.2.5.
4. Using local Kronecker-Weber, show that

$$\mathbb{Z}_p^\times \times \widehat{\mathbb{Z}} \cong G_{\mathbb{Q}_p}^{\text{ab}},$$

and that the filtration on  $\mathbb{Z}_p^\times$  induced by the upper numbering filtration on  $G_{\mathbb{Q}_p}^{\text{ab}}$  coincides with the filtration by the subgroups  $U^{(n)} = 1 + p^n\mathbb{Z}$ .

5. Let  $L/K$  be a finite extension of local fields of characteristic 0. Prove that  $\text{Norm}_{L/K}(L^\times)$  is an open subgroup of  $K^\times$ .
6. Repeat the previous exercise for characteristic  $p > 0$ . (Hint: it may help to reduce to the case of an Artin-Schreier extension; see CFT, exercise 9.4.)
7. See SageMathCloud for this problem.