Math 204B (Algebraic Number Theory), UCSD, winter 2015
Problem Set 7 (due Wednesday, March 4)

Solve the following problems, and turn in the solutions to *four* of them. Note: LMFDB means the L-Functions and Modular Forms Database (`http://www.lmfdb.org`), while CFT means my class field theory notes (`http://math.ucsd.edu/~kedlaya/math204b/cft-overall.pdf`). No SMC component this week, but you may find it helpful for several of the regular problems, so I've provided a dummy folder on SMC in which to submit your work.

1. Do Neukirch, Exercise III.2.2 (which I also explained in class). Then translate this into a formula for the discriminant of L/K written in terms of the *upper* numbering filtration.

2. Let $K = \mathbb{F}_p(t)$ and let $L = K[z]/(z^p - z - t^{-m})$ for m a positive integer not divisible by p. Compute the different of L over K, and show in particular that the exponent of the unique prime above t in the different can be arbitrarily large even though the ramification index is equal to p.

3. Find examples of number fields K of degrees 3, 4, 5, 6 such that for some prime p, K has both a prime ideal ramified over p and a prime ideal unramified over p. Please specify in as much detail as possible how you found your examples (e.g., LMFDB, Sage exhaustive search, explicit construction, etc.).

4. Recall that the Hasse-Arf theorem states that the upper-numbering ramification breaks of any abelian extension of local fields are all integers. Find, with proof, an example in LMFDB of an extension of $\mathbb{Q}_2$ with Galois group equal to the dihedral group of order 8 where one of the ramification breaks is not an integer. (You may also use another source to construct your example, such as Serre's *Local Fields*, but in that case you must still locate your example within LMFDB for credit.)

5. Let K be a number field of degree n with absolute Galois group S_n and Galois closure L. (That is, L is the Galois closure of K over $\mathbb{Q}$, and $\mathrm{Gal}(L/K) = S_n$.) Let D be the absolute discriminant of K, and assume that D is not a perfect square. Prove that L is an everywhere unramified extension of either $\mathbb{Q}(\sqrt{D})$ or $\mathbb{Q}(\sqrt{-D})$, with an explanation of how to choose the sign based on the real and complex embeddings of K.

6. CFT, exercises 8.1 and 8.2 (these count as one problem).

7. CFT, exercises 8.3 and 8.4 (these count as one problem).