1. Let $R$ be the ring of germs at $0$ of meromorphic functions in the complex plane, equipped with the exponential valuation given by measuring the order of vanishing at $0$. Prove that $R$ is a henselian field which is not complete; what is its completion?

2. Let $K$ be a field of characteristic $p$ which is separably closed and complete with respect to a nonarchimedean absolute value. Prove that $K$ is also algebraically closed. (Hint: you may use the exercise from the previous homework about continuity of the roots of a polynomial with respect to the coefficients.)

3. Prove Krasner’s lemma: let $K$ be a field complete with respect to a nonarchimedean absolute value $|\cdot|$. Let $L$ be an algebraic closure of $K$. Let $\alpha_1, \ldots, \alpha_n \in L$ be the roots of some polynomial over $K$. If $\beta \in L$ satisfies

$$|\alpha_1 - \beta| < |\alpha_1 - \alpha_i| \quad (i = 2, \ldots, n),$$

then $K(\alpha) \subseteq K(\beta)$ as subfields of $L$.

4. Let $K$ be the algebraic closure of a field which is complete with respect to a nonarchimedean absolute value. Prove that the completion of $K$ is again algebraically closed. (Hint: use Krasner’s lemma.)

5. Give an example of each of the following. In each case, you need only consider one value of $p$, and it need not be the same across the cases.

   (a) A reducible polynomial over $\mathbb{Q}_p$ whose Newton polygon is a straight line.
   (b) A polynomial over $\mathbb{Q}_p$ whose Newton polygon has integer slopes, but which nonetheless has no roots in $\mathbb{Q}_p$.
   (c) A polynomial over $\mathbb{Q}_p$ which is irreducible of degree 4 with slope $1/2$. (This shows that the denominator of the slope need not equal the degree.)

6. Derive the Eisenstein irreducibility criterion using Newton polygons, then give an example of a new irreducibility criterion that applies in some cases where the Eisenstein criterion fails.