Cohomology of finite groups: abstract nonsense

For scheduling reasons, this lecture is being recorded on Thursday, January 14. For those watching live, as usual there will be 30 minutes of office hours immediately afterward.

HW 9 is due today. Please ask in advance (i.e., today) if you need an extension.

HW 10 is posted. It is due Thursday, January 21.

Nolemes in the has a Maday Jaran 18.

<u>G-modules</u> (et & te & (timte) grap Aclin (right) A & mode i a stella grap A equipped and a dachon (aright). **G-modules** 6x A -> A (9, a) -> a? (at) 2= 25 + 25 i. a right 260-mill les a honomorphism A homomorphism of 6-modules is a honomorphism Q: A-713 of such is al which is 6-equiliant Q: A-713 of such is al which is foregoing at wh

Invariants (and coinvariants)

M= 6-modile. M⁶= L m E M: m⁹= m VSEG <u>Finvalat</u> = mammal subject of M which is G-Fred Mc = M/svonable recetet by = G-coincepts = marinal gubet 41 tren when is 6-fixed.

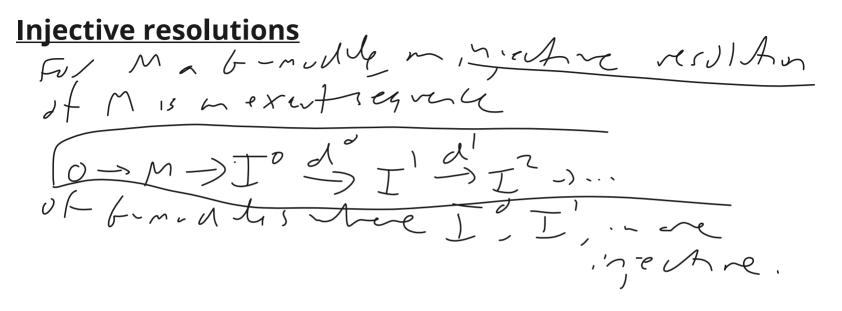
Invariants, coinvariants, and exact sequences let d-Mind Marin DU le n<u>exact sequence</u> (ie. Kenel steach mp = image of previous mg) ir. & -numints 0-) M'G-)MG->M"G is exact fr-ch-For Gomid Mes RXh 0->7/12->4/27>1/12->0 a some e of 6 - mod los mere 6= 2/12 a = a(14/2). to abelia) Jett exact Simily, G-ruinmand fuck 's right exact.

Where we are headed: derived functors

we we lover, dr: the east Michos 1-1'(G, P): 4(-mulles) of abdul 5mg) 1-1°(GA)=AC ishtand (=0,1,2,... such that given a most segence that $() \rightarrow M' \rightarrow M' \rightarrow M' \rightarrow V$ St he yact requerce 0->1-1'(G,M')->14"(6,M)-)~"(G,M')50 (>1-1(6, m)-)H'(6, M) >H'(6, M")>5" 5 H2 (GM) -).

Injective G-modules (Priecher) A 6-mille is Injecher Floren richten AC13 ut komendels, eng Gondlehumenghum A -M Can be extended to 13 The Englandle is is Nobjecto tour nache. (That is, he wheng it 6-model) PEBr 6=4e), mechne - divisite (it news) Insend, Mars Hung (200), M and divisite where many is no justice of ab swar and divisite

The category of G-modules has enough injectives



Definition of the derived functors

Apply 6-inverts: 1 set grend W-M-JEXNEST, 5 d' ... OF alelia Li > 0 Mich is Edge X, dur J'=0 6 A at reassing prust. Define Hills MI- ber(d')/in(di-) (me d'=zermpinho I°, 5, h°(6, M)= M6 (up ho unique Clam: H'(G m) repeasely on h sumphism)

Functoriality let tim whe amonghan of 6-montes NEM, MANTY ryph h fill in vertical Apry G-, huritants: $\begin{array}{c} & M^{6} \rightarrow I^{0} \stackrel{o}{\rightarrow} \stackrel{T^{16}}{\longrightarrow} \stackrel{o}{\rightarrow} \stackrel{i}{\longrightarrow} \stackrel{i}{\rightarrow} \stackrel{i}{\rightarrow} \stackrel$

<u>Key tool: the five lemma</u> Lenne: Cristen diagren of Goodle with exact $\rightarrow M' \rightarrow M \rightarrow M'' \rightarrow D$ $\delta \rightarrow N' \rightarrow N' \rightarrow D$ r ~ (then f is h is uning him (=) f' f' we is uning his mis. more seeally, have some fring sim, la 4 M°-) M'-) m2 - m3 - my Je Je Je Je Je Je Je Je Lether N° -) N' - M2 - MY f e tactory

Key tool: the snake lemma Um Aak u st b-modules lenna Konna diajon $0 \rightarrow M' \rightarrow M' \rightarrow U$ $LF'' \rightarrow U$ $C \rightarrow N' \rightarrow V \rightarrow V$ inthe oractions shsts in exit seguence The tree $d \rightarrow be(f) \rightarrow be(f) \rightarrow be(f'') \rightarrow be(f$ $Scole(f') \rightarrow (stelf) \rightarrow (stelf) \rightarrow (stelf') \rightarrow 0$ Anne & by "diagon dase".

Acyclic resolutions A 6 mille M is sychic if H'(G, M)= 0 leg. If Minjectine, the ayche: Aiso. ULE K-M -M -JO-JO-...) Key MA; IF 0->M -> I' -> I' -> ... IS Law Mic (.x. a exact seque and I, II, ... Gay N.c.) $\frac{1}{16} \left(\frac{6}{16} \right) = \frac{1}{16} \left(\frac{1}{16} \right) = \frac{1}{16} \left(\frac{1}{16} \right) \left(\frac{1}{16} \right)$ the Tic AiG -(ih)G