1. Let $G$ be a finite group and let $H$ be its commutator subgroup. Assuming the existence of the transfer homomorphism $V : G^{ab} \to H^{ab}$ as defined in CFT Theorem 2.3.7, prove that it is zero in this case. (Hint: see CFT exercise 2.3.6.5.)

2. Find the ray class field of $\mathbb{Q}(i)$ of conductor $(3)$ and verify Artin reciprocity explicitly in this case.

3. Let $D$ be an odd squarefree positive integer greater than $3$. Give a formula for the order of the generalized ideal class group $\text{Cl}^m(\mathbb{Q}(\sqrt{-D}))$ in terms of the class number of $\mathbb{Q}(\sqrt{-D})$ and the factors of $D$ and the modulus $m$.

4. Here is an example to illustrate the difference between Dirichlet density and natural density, albeit not for primes. Let $S$ be the set of positive integers whose decimal expansion begins with $1$.

   (a) Prove that $S$ does not have a natural density, in the sense that
   \[
   \lim_{X \to \infty} \frac{1}{X} \#(S \cap \{1, \ldots, X\})
   \]
   does not exist.

   (b) On the other hand, prove that $S$ has a Dirichlet density in the sense that
   \[
   \lim_{s \to 1^+} \frac{\sum_{n \in S} n^{-s}}{\sum_{n=1}^{\infty} n^{-s}}
   \]
   exists, and compute this value.

5. Just for fun, here is another result due to Chebotarëv. Let $p$ be a prime.

   (a) Let $f \in \mathbb{F}_p[x]$ be a nonzero polynomial of degree less than $p$. Prove that the multiplicity of any nonzero element of $\mathbb{F}_p$ as a root of $f$ is less than the number of nonzero coefficients of $f$. (Hint: compare $f$ and $f'$.)

   (b) Let $M$ be the $p \times p$ matrix given by
   \[
   M = (\zeta_p)^{p-1}_{i,j=0}.
   \]
   Prove that every minor of the matrix $M$ is nonzero. (Hint: reduce modulo $1 - \zeta_p$ and apply (a).)