Math 204B (Number Theory), UCSD, winter 2021
Problem Set 14 – due Thursday, February 18, 2021

1. Let K be a finite extension of $\mathbb{Q}_p$. Prove that the intersection of the groups $(K^*)^n$ over all positive integers n is the trivial group.

2. Let $L/K/\mathbb{F}_p((t))$ be finite separable extensions. Prove that $\operatorname{Norm}_{L/K} L^*$ is an open subgroup of K^*. (Hint: reduce to the case of a cyclic extension of prime degree. For the case where $[L : K] = p$, write L as an Artin-Schreier extension and then compute explicitly.)

3. Let p be an odd prime and put $K = \mathbb{Q}_p, L = \mathbb{Q}_p(\sqrt{p})$. Compute the groups $H^i_T(\operatorname{Gal}(L/K), \mathcal{O}^*_L)$ and see that they are nontrivial (by comparison with CFT Corollary 4.2.6).

4. Show that the hypotheses of abstract class field theory (i.e., the class field axiom and the conditions on a henselian valuation) are satisfied in the following case:
   - k is a finite field (and $\overline{k}$ is its algebraic closure);
   - $d : \operatorname{Gal}(\overline{k}/k) \to \widehat{\mathbb{Z}}$ is the isomorphism taking Frobenius to 1;
   - A is the group $\mathbb{Z}$ with the trivial action;
   - $v : A_k = \mathbb{Z} \to \widehat{\mathbb{Z}}$ is the profinite completion.

5. For p an odd prime, compute the ramification breaks for the lower and upper numbering for the extension $\mathbb{Q}_p(\zeta_{p^2})/\mathbb{Q}_p$, directly from the definitions (i.e., without using local reciprocity). You should get that the breaks in the upper numbering are $1, 2$.