**Math 204B (Number Theory), UCSD, winter 2021**
**Problem Set 15 – due Thursday, February 25, 2021**

1. Let $G$ be a finite group. Let $M$ be an induced $G$-module. Prove directly from the definition of $H^i_T$ that $H^i_T(G, M) = 0$ for $i = 0, -1$.

2. Let $K$ be the splitting field of the polynomial $x^4 + 2x + 2$ over $\mathbb{Q}_2$. Show that in the ramification filtration on $\mathrm{Gal}(K/\mathbb{Q}_2)$, the largest break for the upper numbering occurs at $4/3$.

3. Prove CFT Proposition 5.2.6 (the compatibility of the reciprocity map with restriction to a subfield).

4. Prove that the subset $[0, 1] \times \prod_p \mathbb{Z}_p$ of $\mathbb{A}_\mathbb{Q}$ surjects onto $\mathbb{A}_\mathbb{Q}/\mathbb{Q}$.

5. Let $K$ be a number field. Using the finiteness of the class group and the finiteness of the unit group, prove that there exists some $c > 1$ such that for every idèle $x$ in $I_K$ of norm 1, there exists $\alpha \in K^*$ such that every component of $\alpha x$ has norm in $[c^{-1}, c]$.