1. Let $L/K$ be a Galois extension of number fields with Galois group $G$.

   (a) Give an explicit example for which $\text{Cl}(K) \to \text{Cl}(L)^G$ fails to be injective. (Hint: use the principal ideal theorem.)

   (b) Give an explicit example for which $\text{Cl}(K) \to \text{Cl}(L)^G$ fails to be surjective. (Hint: take $L$ to be a quadratic extension of $\mathbb{Q}$ with $\text{Cl}(L) \cong \mathbb{Z}/4\mathbb{Z}$.)

2. Put $K = \mathbb{Q}(\sqrt{13}, \sqrt{17})$. Show that $5^2 \in \text{Norm}_{K_w/\mathbb{Q}_v} K^*_w$ for every place $w$ of $K$ lying above a place $v$ of $\mathbb{Q}$, but $5^2 \notin \text{Norm}_{K/\mathbb{Q}} K^*$.

3. Prove the following special case of the Hasse-Minkowski theorem. Let $K$ be a number field and choose $a, b, c \in K^*$. Suppose that for each place $v$ of $K$, there exist $x, y, z \in K_v$, not all zero, such that $ax^2 + by^2 + cz^2 = 0$. Then there exist $x, y, z \in K$, not all zero, such that $ax^2 + by^2 + cz^2$. (Hint: show that $-c$ is a norm from $K(\sqrt{-b/a})$ to $K$.)

4. (a) Let $K$ be a field of characteristic not equal to 2. Show that 16 is an 8th power in $K$ if and only if one of $-1, 2, -2$ is a square in $K$.

   (b) Put $K = \mathbb{Q}(\sqrt{7})$. Show that 16 is an 8th power in every completion of $K$, but not in $K$ itself.

5. Let $K$ be a number field, let $S$ be a finite set of finite places of $K$, and let $m$ be a positive integer. Prove that there exists a cyclic extension $L$ of $K$ contained in $K(\zeta_n)$ for some $n$ such that for all $v \in S$, for some place $w$ of $L$ above $K$, $[L_w : K_v]$ is divisible by $m$. 