Math 204B (Number Theory), UCSD, winter 2025
Problem Set 3

1. CFT notes 2.1, exercise 1.

2. Let K be a number field. Prove that any number field containing L with class number 1 contains the Hilbert class field of L. (Hint: look at the compositum of L with the Hilbert class field.) Then deduce as a corollary that the class field tower of K stabilizes if and only if K is contained in some number field of class number 1.

3. Let K be a number field and let $\mathfrak{m}$ be a modulus of K. Prove that the ray class field of K with modulus $\mathfrak{m}$ is unique up to isomorphism. (Hint: see CFT notes 2.4, exercise 6.)

4. Let K be a number field. Prove that the set of primes of K of absolute degree greater than 1 has Dirichlet density 0. (The example $K = \mathbb{Q}(i)$ was discussed in lecture on January 27.)

5. CFT notes 2.4, exercise 7.