A CONVERSATION ABOUT THE PUTNAM COMPETITION

The following is an edited transcript from the Putnam MathJam held on Art of Problem Solving on December 18, 2017. This was a moderated question-and-answer session hosted by Kiran Kedlaya, featuring two past members of the Questions Committee: David Savitt (2013–2015) and Byron Walden (2002–2004, 2015–2017). Thanks to Jeremy Copeland for setting up the session.

In the transcript, each statement is labeled **A**, **S**, **W**, **K** denoting respectively the audience, Savitt, Walden, or Kedlaya. Questions from the audience are also indicated by *italics*. Some questions and answers have been lightly edited for typography and/or rearranged to form a more coherent sequence.

Introductions.

K To begin with, let me start by asking David and Byron to say a few words about how and why they came to be involved with the Putnam.

S Like most or all of you, my involvement with the Putnam started as a student. I enjoyed the problems enough that I kept up with them while I was a grad student, and then got involved with Putnam training at the institutions where I've been a faculty member. It was a big honor (and something of a terrifying responsibility!) when I was asked to join the problem committee.

W I was coaching the Putnam team at Santa Clara University and writing our high-school and first-year math contests. The first go-round in 2002 several folks who were asked couldn't join the committee and the director asked me to jump in.

The Questions Committee.

A *Who proposes Putnam problems?*

S There isn't really a proposal process. There's a committee (three people each with staggered 3-year terms) and they write the problems.

K How is the committee itself chosen?

S Each year one spot opens up on the committee, and the directors invite someone new. (I guess they keep a list of possibilities in mind.) One thing that surprised me is that they try to have some geographical distribution: one person from the east coast, one from the west coast, and one in between.

A *Did everyone on the writing committee actually take the Putnam as a student?*

S It's not a requirement to have done the Putnam as a student, and certainly people who were students outside North America have been on the committee.

K I gather many of the committee members from outside North America have experience with math competitions in their own countries and/or the IMO.

A *Can outsiders propose problems?*

S Essentially no. The directors are rightly paranoid about security. I took suggestions from people I knew and trusted. (Such as Kiran!)

A *Have there been security incidents in the past?*

S If there were major security incidents in the past I'm afraid that's not something I would know (as just a committee member, not a director). There is a clear directive to everyone involved to be mindful of security, however, for whatever reason.

Composing problems for the Putnam.

A *How do you come up with the questions for the Putnam?*

S I think this is different for everyone. I know some people who can decide they want to sit down and write a problem, and do it. That isn't me. I had a file of potential problems I'd come across over the years while doing research or reading papers, and then when I was on the committee I sort of went into overdrive keeping my eyes open for things. Many of the problems I wrote were lemmas from papers that I either wrote or read, or special cases thereof, or variants.

W For me, an example was 2017A5. I just tried to come up with a game that might yield an interesting problem. New games are good since they have a good chance of being novel. With the card game, I worked out what the probabilities would be. My first take on the question was just showing there was never a case where all three players had the same chance of winning (easy induction). But since I had worked it all out, the committee decided the full result was more interesting and made it A5.

A *Do you have any suggestions on writing quality problems?*

S Write a lot of problems and then throw away the bad ones. (-: That's only half facetious. You have an idea for a problem, maybe some nifty argument that you'd like to wrap into a problem, but that wrapping is key. Sometimes you do have to let things go if you can't quite get it to work.

A *How do you figure out if the problems you create are too difficult, not difficult enough, or Putnam-quality?*

S It's a bit hard to judge your own problems for yourself. It's so important that the exam is written by a group of people (nominally 3 people, but there's actually a fourth person[1] who helps manage the committee and provides some continuity). We all solve each other's problems and give two ratings: a difficulty rating and an enjoyability rating. When we meet to write the exam we compare and come to a consensus.

A *Given that you are all professors, how do you judge the difficulty of a problem you attempted given your experience with those of a student?*

S For me, I've "taken" the Putnam pretty much every year since 1992, so I think I have some continuity in my judgment of the problem difficulty. It's true that what arguments seem obvious/natural have changed over time for me, but I believe (hopefully correctly) that I'm self-aware about that.

K (ditto David's answer for me)

A *About how many problems do you actually write, and what are the chances for one to actually be on the test?*

S The committee has three people, so you're hoping to contribute four problems per year, which means writing maybe 8+ problems and having half of them get picked. In practice you tend to run out of energy a bit in your third year on the committee, so maybe it's more than that in years 1 and 2 and fewer in year 3.

A *Wait, people only get 3 years on the committee?*

S Right. It's a lot of work. 3 years is plenty!

K Byron has done it twice, but with a gap of almost 10 years in between.

A *Will the committee ever run out of problems to choose from in the foreseeable future?*

S The great thing is that there's constantly new people on the committee and no shortage of mathematics!

A *Have there been any case that a Putnam problem appeared in a previous math contest?*

W Yes, there are so many competitions these days that it's really hard not to duplicate. If we know about it existing elsewhere, we don't use it.

S It's always an accident (in the instances I know about). One problem is, again for security reasons you can't show the exam to lots of people, so it's somewhat hard to catch.

W I was worried about A2 this year being known, and I nosed around a bit through journals, but it turned out to have been in the Indian Olympiad a few years back.

A *Any favorite problems?*

K I feel a bit like asking someone to choose their favorite problem is like asking a parent to choose a favorite among their children...

S OK, I guess I can cop to being responsible for working the Putnam Quarter Quell into the 75th Putnam (problem 2014B5).

K I guess that also makes you responsible for using names that made it tricky to use male/female pronouns in the solution to that problem.

S To me that was a feature rather than a bug.

A *Have you ever made a problem, and either a contestant or another committee member found a solution significantly different than what you had?*

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[1]This refers to the Asssociate Director of the competition, Mark Krusemeyer (Carleton College).

S Definitely. One example for a problem I wrote was Elkies's solution to 2015A6. For that problem, what I wondered ahead of the competition was whether some someone was going to find a clever way to reduce to the case where the matrix M is invertible, but instead the surprise was something else.

Topics.

A *What university level courses should complete before being able to attempt every question? In other words, what topics does the Putnam exam cover?*

K I don't believe there is an official syllabus, but generally the topics can include everything admissible for the USAMO, plus calculus (and a bit of real analysis, like point-set topology), linear algebra, and abstract algebra.

A *When the committee chooses problems for Putnam, do they make sure to include at least one problem in each topic (such as linear algebra, calculus, number theory) or just choose the best problems?*

S Yes, the committee tries to balance it out. Sometimes you wish you had another good problem in some area and you just don't — in that case you certainly pick a great problem in an over-represented area ahead of a so-so problem in an under-represented one. The committee does try to identify those needs in advance and try to come up with more, sometimes successfully and sometimes not. Part of this is balancing the mathematical interests of people on the committee. If there's no algebraist on the committee, you'll probably be short of algebra problems.

A *What are some differences between the Putnam and high school math exams in terms of difficulty and style?*

K Besides the fact that the Putnam assumes more background (see above), one big difference is the time control: Putnam problems are meant to be solved in an average of 30 minutes apiece, so they tend not to have multiple moving parts (except maybe A6/B6). There is usually one key idea, and finding that (and expressing it clearly and correctly) is typically the difference between an 0/1/2 score and an 8/9/10 score. The distinction among 8/9/10 is down to execution (not making any minor errors).

A *Is it possible for there to be easier questions on Putnam than USAMO?*

S This is somehow very personal. As a student I found Putnam problems more suited to me than USAMO problems. But certainly there are USAMO winners who run into the Putnam and don't do as well.

W Answering the question about determining difficulties, it's really very subjective. We work each others' problems and do our best. There was a lively discussion at the grading session about whether A5 or A6 was harder this year. Since I wrote A5 and couldn't do A6, I think we had it right, but others saw it differently.

A *How well would you say that undergraduate classes prepare you for the Putnam exam? What about compared to high school competitions and high school classes?*

S In writing the exam, the hope is that all the problems are solvable without too much specialized background, but certainly calculus and linear algebra are all fair game. To take group theory as an example, it's fine for a problem to require knowing what a group is, but not any specialized knowledge of group theory.

A *What would you recommend for students without any Olympiad background? Obviously, IMO, US-AMO students would be more successful than those who didn't write any Olympiads at all. Any tips for latter people?*

S Responding to the premise here... I think this has actually changed a bit over time. My sense, anecdotally, is that you see the Putnam Fellows (top 5) more dominated by IMO folks now than 20 years ago. I could be wrong about that, but certainly you see more 3–4 time winners than one used to.

K Re the correlation between IMO and Putnam: one change that I noticed starting in the mid-1990s is that before then, the strongest math students coming to the US for graduate school had done undergraduate degrees in their countries of origin. Since then (and up to now), it is far more common for such students to come to the US directly as undergraduates, thus making them eligible to participate in the Putnam.

Grading.

A *How are Putnam problems graded?*

K I can help with this one, having done grading for the Putnam. There are two grading sessions, one for the A set, one for the B set (these take place a week or two after the exam). Each problem is graded by a set of 2 or 3 people (with A1/A2/B1/B2 usually getting 3 people since more people submit those problems). Only the scores 0,1,2,8,9,10 are used (this is a pseudo-official rule; it is very rarely violated).

W A quick answer about scoring. Scores of 3 to 7 points on a question are no longer disallowed, but they are discouraged. There will be some this year (2017) but not very many.

A *While it is generally a pseudo-rule, why aren't the scores 3 through 7 given?*

K I don't have an authoritative answer for this, but my understanding is that excluding the middle is meant to make it easier for the graders. Going through 3000 papers in a weekend is a pretty daunting task (by comparison, grading the USAJMO/USAMO involves about 500 papers in a similar amount of time) and the more options you have per paper, the longer each decision is likely to take.

A *In the past, there has often been an air of mystery regarding Putnam grading, with some people feeling that they get docked off for points "unnecessarily" with solutions they feel are correct/almost correct (regardless of whether or not this feeling is actually valid). When grading Putnam problems, what sorts of things do you generally look for, and how do you distinguish between a solution receiving a 1 and a solution receiving a 9?*

S This is a great question. I wondered it myself as a student (there was one year where my score was very different from what I was expecting), and I'm not totally sure that I have a satisfactory answer now that I've been on the grading side of things. One thing to say is that grading is a small operation so there's not a huge amount of time to scrutinize solutions for points. In particular on problems that huge numbers of people have attempted (A1,A2,B1,B2) there's not much tolerance for an incorrect final answer. There's more margin for error on problems 3–6. I tend to think 9 is more-or-less a full solution with either a small calculation error towards the end or a minor (not major) bit of justification missing. In any case there is a very carefully prepared grading key, which undergoes some changes as new solutions crop up, but not much.

K There is also a second round of grading later for the high scorers (generally the top 200), in order to catch critical errors in the grading process; but that generally involves just the Putnam directors and so I have never been privy to it.

A *What kind of people (i.e., their mathematical background) grade the tests?*

S It's been a while since I graded, but my recollection is that essentially everyone is a professor.

K Does the committee generally participate in grading?

S When I came on to the committee, it was suggested that I come to the grading session to see what it's like, so I did that once. Now I'm on a list of people that get asked every year, but I haven't been able to go back. (The second year my daughter was born at around grading time, and now I usually have family obligations around then, like my daughter's birthday!)

A *In general, what results are graders fine with contestants citing (as opposed to proving) in solutions?*

S I can't speak for everyone, but if a theorem has a name, I think you should be fine using it if you use it correctly.

K To be on the safe side, if you are using a fancy theorem, it helps a lot if you use a commonly recognized name for it. Failing that, I would advise giving a full (and correct!) formal statement of the result, so that the grader can see that you aren't just inventing the statement out of wishful thinking.

B For citing results, one common error is to say that the result follows from Schwarzenegger's Theorem or whatever without stating what the hypotheses are and how they sync to the problem.

A *What percent of people solve A6/B6?*

S Depends! I was responsible for a B6 that no one solved.)-: That one surprised me, actually.

K Followup: what percent of people are *supposed* to solve A6/B6?

S Re A6/B6: the committee generally does intend to order the problems in each section by difficulty, so you're imagining just a handful of people will solve A6/B6.

Rules and eligibility.

A *How long has the Putnam been running?*

W The Putnam Exam goes back to 1937. For the first few years of its existence, it was an English competition.

A *What is the origin of the rule that any individual may compete in Putnam at most 4 times?*

S I don't know for sure but I think of it as something like your 4 years of varsity sports eligibility.

A *How does this rule factor into students returning from a leave of absence?*

K The rule is quite simple: no individual may take the Putnam more than four times. This could be at more than one school, or with a gap of years in between.

A *Can high school students take the Putnam?*

S Yes, but if you take it officially it uses up one of your four years of eligibility.[2]

A *So shouldn't people outside the US be able to take Putnam?*

S Canadians[3] can take the Putnam! There are some issues with having it be worldwide, though. One is timing[4] and security.

K There are a small handful of overseas locations for the Putnam, due to various special cases. For instance, the Budapest Semesters in Mathematics[5] program administers the Putnam for its students. In general, the governance of the Putnam is partly limited by the terms of the bequest from the Putnam Foundation. This is one reason why the format has stayed fairly constant over the years (as opposed to, say, the USAMO, which has seen some changes in the time control, number of questions, etc.)

S I will say that the directorship has changed hands in the last year, and those of you who took the exam earlier this month and also last year will have noticed big changes. I don't know what other changes the new director may have in store.

K I would guess that with the change in the directorship, more changes are likely, but it is too early to speculate about that.

A *Is the Putnam available in other languages other than English?*

S No, but I believe it is unofficially permitted for Canadians to submit solutions in French.

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[2]A notable example is Gabriel Carroll, who took the Putnam officially as a high school student in 2000 and was named a Putnam Fellow. He went on to become a four-time Fellow, but was ineligible to compete in his final undergraduate year.

[3]For example, Savitt took the Putnam as an undergraduate at the University of British Columbia.

[4]The official times of the competition vary across time zones; for example, the competition starts at 10AM in the Eastern time zone and 8AM in the Pacific time zone.

[5]Disclosure: Kedlaya serves on the board of directors of Pro Mathematica Arte, the parent organization of the BSM program.