An overview of the $p$-adic local Langlands correspondence (after Colmez)

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Online Conference in Automorphic Forms
virtual conference sponsored by the Rényi Institute
June 1, 2020

Kedlaya was supported by NSF (grant DMS-1802161) and UC San Diego (Warschawski Professorship).
Let $F$ be a global field (a number field or a finite extension of $\mathbb{F}_p(t)$ for some prime $p$). Fix a prime number $\ell$ which is nonzero in $F$.

The global Langlands correspondence for the group $\text{GL}_n$ is supposed to relate continuous representations of the absolute Galois group $G_F$ on $n$-dimensional $\overline{\mathbb{Q}}_\ell$-vector spaces (unramified away from finitely many places of $F$) with automorphic representations of the adelic group $\text{GL}_n(\mathbb{A}_F)$. Under this correspondence, the spectrum of Frobenius at a place $v$ of $F$ on the Galois side is supposed to match the spectrum of the Hecke operator at $v$ on the automorphic side.

The case $n = 1$ reproduces class field theory.
The global Langlands correspondence

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The case $n = 1$ reproduces class field theory.
Let $F$ be a local field (a complete discretely valued field with finite residue field). Fix a prime number $\ell$ which is nonzero in the residue field of $F$.

The local Langlands correspondence for the group $\GL_n$ relates continuous representations of $G_F$ on $n$-dimensional $\overline{\mathbb{Q}}_\ell$-vector spaces (with no condition on ramification) with representations of the adelic group $\GL_n(\mathbb{A}_F)$.

There is supposed to be local-global compatibility of the Langlands correspondence: for $F$ a global field and $v$ a place of $F$, restricting from $G_F$ to $G_{F_v}$ on the Galois side is supposed to correspond to restricting from $\mathbb{A}_F$ to its factor $F_v$.

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The local Langlands correspondence ($\ell \neq p$)

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Again, the case $n = 1$ reproduces class field theory.
Let $F$ be a local field of residue characteristic $p$.

One can ask whether there is a meaningful version of the local Langlands correspondence in this setting which exhibits local-global compatibility. Because there are “many” continuous representations of $G_F$ on finite-dimensional $\mathbb{Q}_p$-vector spaces, one must rigidify this question by asking for compatibility with $p$-adic analytic interpolation.

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\[ \ell = p \]
The case of $GL_2(Q_p)$

A miracle happens for the group $GL_2$ over $Q_p$: Colmez has constructed a correspondence that does everything one would want. (The existence of same had been conjectured by Breuil.)

This uses Fontaine’s theory of $(\varphi, \Gamma)$-modules, which gives a convenient alternate description of the category of continuous representations of $G_{Q_p}$ on finite-dimensional $Q_p$-vector spaces. More on this description shortly.

Reading off the Langlands correspondence from a $(\varphi, \Gamma)$-module is in the same spirit as other constructions in $p$-adic Hodge theory; for example, given the $p$-adic étale cohomology of $X_{Q_p}$ for some smooth proper scheme $X$ over $\mathbb{Z}_p$, one can read off the comparison isomorphism with crystalline cohomology (Berger).
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What is a \((\varphi, \Gamma)\)-module? (version 1)

Let \(A\) be the \(p\)-adic completion of \(\mathbb{Z}_p((\pi))\). This ring admits an endomorphism \(\varphi\), and automorphisms indexed by \(\gamma \in \Gamma = \mathbb{Z}_p^\times\), characterized by

\[
\varphi(1 + \pi) = (1 + \pi)^p,
\]

\[
\gamma(1 + \pi) = (1 + \pi)^\gamma = \sum_{n=0}^{\infty} \frac{\gamma(\gamma - 1) \cdots (\gamma - n + 1)}{n!} \pi^n
\]

plus continuity for the inverse limit topology given by putting the \(\pi\)-adic topology on \(\mathbb{Z}/p^n\mathbb{Z}((\pi))\).

A (projective, étale) \((\varphi, \Gamma)\)-module over \(A\) is a finite free \(A\)-module \(M\) equipped with commuting semilinear continuous actions of \(\varphi\) and \(\Gamma\), for which the induced map \(\varphi^*M \to M\) is an isomorphism.
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Theorem (Fontaine)

There is an explicit equivalence of categories between the category of \((\varphi, \Gamma)\)-modules over \(A\) and the category of continuous representations of \(G_{\mathbb{Q}_p}\) on finite free \(\mathbb{Z}_p\)-modules.
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What is a \((\varphi, \Gamma)\)-module? (version 2)

Let \(A^\dagger\) be the subring of \(A\) consisting of series which converge in some region of the form \(* < |\pi| < 1\) (such series are said to be overconvergent); this subring is stable under \(\varphi\) and \(\Gamma\). Define a \((\varphi, \Gamma)\)-module over \(A^\dagger\) using the same recipe as over \(A\).

Theorem (Cherbonnier–Colmez)

The categories of \((\varphi, \Gamma)\)-modules over \(A^\dagger\) and \(A\) are equivalent via base extension. In particular, by Fontaine they are both equivalent to the category of continuous representations of \(G_{\mathbb{Q}_p}\) on finite free \(\mathbb{Z}_p\)-modules.
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A monoid algebra reinterpreted (part 2)

It is natural to view \( \varphi \) and \( \Gamma \) together as forming a single commutative monoid isomorphism to \( \mathbb{Z}_p \setminus \{0\} \), acting on \( A \) and \( A^\dagger \) as

\[
x(1 + \pi) = (1 + \pi)^x.
\]

Consequently, any \( (\varphi, \Gamma) \)-module may be viewed as a left module for the twisted monoid algebra \( A\langle \mathbb{Z}_p \setminus \{0\} \rangle \).

Thanks to the continuity condition, we can extend the action of \( A\langle \mathbb{Z}_p \setminus \{0\} \rangle \) to a larger ring.* Namely, identify the Iwasawa algebra \( \mathbb{Z}_p[1 + p\mathbb{Z}_p] \) with \( \mathbb{Z}_p[T] \). We then define a ring structure on \( \mathbb{Z}_p[\pi, T] \) so

\[
(1 + T)^{-1}(1 + \pi)(1 + T) = (1 + \pi)^{1+p},
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invert \( \pi \), impose the \( \pi \)-adic topology modulo each power of \( p \), and take the inverse limit.

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The same ring can be desired from the untwisted monoid algebra over $\mathbb{Z}_p$ for the monoid

$$\left( \mathbb{Z}_p \setminus \{0\} \quad \mathbb{Z}_p \right)$$

by identifying

$$\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} \text{ with } (1 + \pi)^x.$$
Automorphic representations from \((\varphi, \Gamma)\)-modules

As noted above, from a \((\varphi, \Gamma)\)-module \(M\) we obtain an action of the monoid
\[
\left( \mathbb{Z}_p \setminus \{0\} \quad \mathbb{Z}_p \\
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Let \(\psi : M \to M\) be the reduced trace\(^\dagger\) of \(\varphi\). By replacing \(M\) with
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we obtain an object with an action of the group
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By a suitable induction from this subgroup (the \textit{mirabolic}) to \(GL_2(\mathbb{Q}_p)\),
we obtain Colmez’s candidate for the Galois-to-automorphic construction.
(One checks that this works by looking carefully at certain specializations.)

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Overconvergent descent and locally analytic vectors

As noted earlier, a \((\varphi, \Gamma)\)-module over \(A\) descends canonically to the subring \(A^{\dagger}\). By tracing this descent through Colmez’s construction, we obtain a subrepresentation of \(GL_2(\mathbb{Q}_p)\).

This turns out to be the \textbf{locally analytic} vectors in the original representation. The key point is that \(A^{\dagger}\), while dense in \(A\) for the inverse limit topology, admits an alternate topology under which \(A^{\dagger}\) is complete and the action of \(\Gamma\) remains continuous.
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Okay, now what?

So far so good, but how do we go past GL$_2(\mathbb{Q}_p)$?

If $F$ is a finite extension of $\mathbb{Q}_p$, we can exhibit a similar theory of $(\varphi, \Gamma)$-modules associated to representations of $G_F$. However, this fails to reproduce the previous success for two reasons.

- The base ring will be a finite étale algebra over the previous one. In particular, it will still correspond to completing a group algebra for a commutative $p$-adic Lie group of rank 1 over $\mathbb{Q}_p$ (not $F$).

- The group $\Gamma$ will be replaced by a subgroup of finite index. In particular, it will remain a $p$-adic Lie group of rank 1 over $\mathbb{Q}_p$ (not $F$).

Similar issues arise if we try to replace GL$_2$ with a group of higher rank, such as GL$_n$. 
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Beyond $GL_2(Q_p)$

In order to go further, we need additional constructions of multivariate $(\varphi, \Gamma)$-modules associated to Galois representations.

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The action of $\Gamma$ in the usual theory of $(\varphi, \Gamma)$-modules is derived from the action of $\mathbb{Z}_p^\times$ on $\mathbb{Q}_p(\mu_{p\infty})$. One can construct a parallel theory for any infinitely ramified $p$-adic Lie extension of any finite extension of $\mathbb{Q}_p$ except that this happens in the language of perfectoid rings, which does not provide access to locally analytic vectors.

To do that, one must establish variants of the Cherbonnier-Colmez overconvergent descent. Recent progress has been made on this by H. Gao and T. Liu.

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For more information...


To be continued next summer in Budapest!